

Name of Course	: <b>CBCS B.Sc. (H) Mathematics</b>
Unique Paper Code	: <b>32351102_OC</b>
Name of Paper	: <b>C2-Algebra</b>
Semester	: <b>I</b>
Duration	: <b>3 hours</b>
Maximum Marks	: <b>75 Marks</b>

*Attempt any four questions. All questions carry equal marks.*

1. Find the seventh roots of unity. Use that to solve the equation

$$(z+1)^7 = z^7(\cos 7\theta + i \sin 7\theta)$$

2. Explain the difference between Principle of Mathematical Induction (usual form) and strong form of Principle of Mathematical Induction, if any. Apply Principle of mathematical induction to prove the following:

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + \dots + \frac{1}{(3n+1)(3n+2)} = \frac{n}{6n+4}$$

3. Find the domain and range of the function  $f(x) = -2 \log_2(x-1)$  of a real variable. Determine whether the function is one-to-one and onto. Find a formula for  $f^{-1}$ , if exists.
4. Find the general solution to the linear system of equations

$$x_1 + x_2 + 2x_4 - 3x_5 = 4$$

$$x_1 + x_2 + x_3 + 2x_4 - 3x_5 = 3$$

$$4x_1 + 2x_2 + 4x_3 - 6x_5 = 8$$

$$4x_1 + 3x_2 + x_3 + x_4 + 9x_5 = 9$$

by row reducing the matrix to Echelon form. Encircle the leading entries; list the basic variables and free variables. Write the general solution in parametric vector form.

5. Let  $A = \begin{pmatrix} 1 & 0 & 2 \\ -2 & 1 & -6 \\ 3 & -2 & 4 \end{pmatrix}$ ;  $b = \begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$ . Define a linear transformation  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  $T(x) = Ax$ .

Find a vector  $x$  whose image under  $T$  is  $b$ . Determine whether the vector  $x$  is unique or not.

6. Find the rank of the matrix  $A = \begin{pmatrix} 1 & -4 & 7 & 4 \\ 1 & -1 & 3 & -3 \\ 2 & 3 & 0 & -2 \\ 4 & 1 & 5 & -9 \end{pmatrix}$ .

Find the basis and dimension for Null space of  $A$ .